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# A study of ${ }^{42} \mathrm{Sc}$ using the ${ }^{40} \mathrm{Ca}(\alpha, \mathrm{d}){ }^{42} \mathrm{Sc}$ reaction 

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#### Abstract

States of $T=0$ in ${ }^{42} \mathrm{Sc}$ were populated via the ${ }^{40} \mathrm{Ca}(\alpha, \mathrm{d}){ }^{42} \mathrm{Sc}$ reaction at a bombarding energy of 25.5 MeV . Deuteron angular distributions were measured to the states at $0.61 \mathrm{MeV}\left(1^{+}+7^{+}\right), 1.491 \mathrm{MeV}\left(3^{+}\right)$and $1.511 \mathrm{MeV}\left(5^{+}\right)$. The relative strength of states up to an excitation of 3.03 MeV were measured and compared with the predictions of a model which considered excitation of the ${ }^{40} \mathrm{Ca}$ core. Reasonable agreement is found for states up to 2.27 MeV . The results are consistent with the 1.846 MeV level being the $J^{n}=3^{+}$head of a deformed band but not with the level at 2.391 MeV being the $4^{+}$member of this band.


## 1. Introduction

According to simple shell-model theory the low-lying states of ${ }^{42} \mathrm{Sc}$ should be composed of a neutron and proton outside an inert core-the so called two-particle ( 2 p ) states. However, it is found experimentally that above an excitation energy of 2 MeV the density of levels far exceeds the number that can be explained on this basis. More recently the study of individual levels shows states, from 1.8 MeV upwards, that are inexplicable on the two-particle picture.

The principal sources of information on excitation energies, spins and parities of levels up to 3 MeV excitation have been the direct interaction work of Sherr et al (1971), Schaeffer (1970) and Puhlhofer (1968), the proton-gamma ray angular correlation measurements following the reaction ${ }^{40} \mathrm{Ca}\left({ }^{3} \mathrm{He}, \mathrm{p} \gamma\right)^{42} \mathrm{Sc}$ (Nicholas et al 1969, Balamuth et al 1970) and the study of the positron decay of ${ }^{42} \mathrm{Ti}$ (Gallmann et al 1969). Lifetime measurements on certain levels in this region have been made by Roberson and Van Middelkoop (1971), Brown et al (1971) and Bertin et al (1972). In order to explain the number and properties of the observed states, additional positive parity states have been proposed by Flowers and Skouras (1969) and Sherr et al (1971) in which a neutron and a proton are excited into the fp shell from the ${ }^{40} \mathrm{Ca}$ core to form levels of four-particle-two-hole ( $4 \mathrm{p}-2 \mathrm{~h}$ ) structure. Similarly negative parity states of three-particle-one-hole ( $3 \mathrm{p}-1 \mathrm{~h}$ ) nature have been calculated by Dieperink and Brussaard (1968).

Comparison of the experimental level scheme with that predicted by Flowers and Skouras (1969) shows good agreement up to 2.5 MeV -the limit of comparison imposed by lack of experimental data. This success of the model allows the prediction of more sensitive observables such as gamma ray lifetimes. The agreement between the available lifetime determinations and those predicted by the model is again good. A different test of the ${ }^{42} \mathrm{Sc}$ wavefunctions, but one of comparable sensitivity to gamma ray lifetimes,
lies in accounting for the relative strengths of states formed in a direct interaction twoparticle transfer process. Here the cross section to any state is composed of a coherent sum of contributions over all allowed single-particle levels for the transferred particles. Such a test is made in the present experiment where the strengths of $T=0$ states of ${ }^{42} \mathrm{Sc}$ were measured and compared with the predictions of calculations incorporating mixed ( 2 p ) and ( $4 \mathrm{p}-2 \mathrm{~h}$ ) wavefunctions. The selection of states of $T=0$ is effected by the ( $\alpha, d$ ) reaction through the conservation of isospin and selection rules which forbid the population of $(\mathrm{f})^{2},(\mathrm{p})^{2}$ or $(\mathrm{fp})$ components of states having $T=1$ and even spin.

## 2. Experimental techniques

Targets of natural metallic calcium of thickness $35 \mu \mathrm{~g} \mathrm{~cm}^{-2}$ and $100 \mu \mathrm{~g} \mathrm{~cm}^{-2}$ were evaporated onto backings of $15 \mu \mathrm{~g} \mathrm{~cm}^{-2}$ carbon and gold foil. These targets were mounted under vacuum and bombarded with a beam of $25.5 \mathrm{MeV} \alpha$ particles from the University of Minnesota tandem generator. An Enge split-pole magnetic spectrometer was used to analyse the outgoing particles which were detected in an array of four position-sensitive surface-barrier counters mounted in the focal plane of the spectrometer. The energy and positional information from these detectors allowed the selection of deuterons from other emergent particles.

The use of a large spectrometer aperture of 1.297 msr and angular width $3^{\circ}$ was dictated by the small cross sections to many ${ }^{42} \mathrm{Sc}$ states. Under these conditions the deuteron groups had full widths at half maximum height of 10 keV for the $35 \mu \mathrm{~g} \mathrm{~cm}{ }^{-2}$ targets and 17 keV for those of $100 \mu \mathrm{~g} \mathrm{~cm}^{-2}$. This was sufficient to resolve the states at 1.491 MeV and 1.511 MeV but insufficient to separate the levels at 0.611 MeV and 0.619 MeV .

The spectrometer system was energy calibrated by adjusting the spectrometer field to step a group of known excitation, the $\left(1^{+}+7^{+}\right)$states in ${ }^{42} \mathrm{Sc}$ taken to lie at 0.619 MeV , across the detector array. The accuracy of ${ }^{42} \mathrm{Sc}$ state energies deduced from these data was estimated by comparing such determinations with those from precision measurement of their gamma decay (Nicholas et al 1969). In this way a standard deviation error of 6 keV was obtained for excitation energies determined in this experiment.

## 3. Results

The angular distributions measured to the states at $0.611+0.619 \mathrm{MeV}, 1.491 \mathrm{MeV}$ and 1.511 MeV are shown in figure 1. In each case the distribution is characterized by the lower value of $L$ allowed in the two-particle transfer. For the levels at 0.61 MeV the distribution is observed to be dominated by $L=6$ transfer to the $0.619 \mathrm{MeV} \mathrm{7}^{+}$state where the $\left(\mathrm{f}_{7 / 2}\right)^{2}$ particles are coupled to correspond to the simple delivery of a deuteron to the ${ }^{40} \mathrm{Ca}$ core.

States at energies greater than 1.6 MeV in ${ }^{42} \mathrm{Sc}$ were contaminated at certain angles by deuterons from the reactions ${ }^{12} \mathrm{C}(\alpha, \mathrm{d}){ }^{14} \mathrm{~N}$ and ${ }^{13} \mathrm{C}(\alpha, \mathrm{d}){ }^{15} \mathrm{~N}^{*}$ (to states at $5 \cdot 28$, 5.31 and 6.33 MeV ). The contamination groups were strong and poorly focused. Their presence obscured regions of about 100 keV of excitation in ${ }^{42} \mathrm{Sc}$ and prevented angular distributions being obtained for levels above 1.6 MeV . For these states, differential cross section measurements from uncontaminated angles were used to evaluate a strength of formation of the state. This was done by dividing the measurements into
the angular ranges $10^{\circ}-24^{\circ}, 25^{\circ}-39^{\circ}$ and $40^{\circ}-54^{\circ}$. Measurements within each range were averaged to give a mean strength and the mean strengths were summed to give the strength of the state. An estimate of the error involved in this simplified method of integration was made by using it on the distributions for the states below 1.6 MeV . It was found that the extreme high and low values that could be generated in this way all lay within $25 \%$ of the accurately integrated value. Although the distributions and integrations do not extend beyond $55^{\circ}$, no additional error in the strengths should result from neglecting further integration as the intensities decrease swiftly with increasing angle and no peaks at large angles are expected.

The energy levels observed are listed in table 1 with their strengths of formation (in arbitrary units). The excitation energies for levels below 2.28 MeV were taken from the precision gamma ray measurements and are accurate to about 1 keV , the energies


Figure 1. Angular distributions to the $1^{+}+7^{+}$states at $0.61 \mathrm{MeV}(\odot)$, the $5^{+}$state at $1.511 \mathrm{MeV}(\mathbf{A})$ and the $3^{+}$state at $1.491 \mathrm{MeV}(+)$. The curves are dwbA fits to the data normalized to each state separately.

Table 1. The energy levels and relative strengths measured in this work

| Energy <br> $(\mathrm{MeV})$ | Strength <br> $(\mathrm{au})$ | Energy <br> $(\mathrm{MeV})$ | Strength <br> $(\mathrm{au})$ | Energy <br> $(\mathrm{MeV})$ | Strength <br> $(\mathrm{au})$ |
| :--- | :--- | :--- | :--- | :--- | ---: |
| $0.611+0.619$ | 740 | 2.391 | 13.8 | 2.832 | 4.7 |
| 1.491 | 90 | 2.439 | 3.5 | 2.846 | 6.8 |
| 1.511 | 296 | 2.459 | 4.2 | 2.883 | 1.1 |
| 1.846 | 3.3 | 2.540 | 3.7 | 2.914 | 20.1 |
| 1.889 | 5.8 | 2.588 | 2.0 | 2.964 | 4.6 |
| 2.189 | 11.3 | 2.653 | 5.0 | 2.997 | 5.2 |
| 2.223 | 12.1 | 2.726 | 3.1 | 3.024 | 14.1 |
| 2.270 | 9.9 | 2.795 | 4.8 |  |  |

of the higher levels were determined in this experiment having an error of 6 keV as discussed above. The strength values of the levels at 1.491 MeV and 1.511 MeV are accurately integrated from the distributions of figure 1 and have an uncertainty of $5 \%$, while those for the higher states are estimated to be accurate to approximately $30 \%$.

## 4. Calculation of the excitation of ${ }^{42} \mathrm{Sc}$ states

The relative strengths of the ${ }^{42} \mathrm{Sc}$ states were predicted using the DWBA expression for the two-nucleon transfer process. The spectroscopic amplitudes were evaluated using wavefunctions for the ${ }^{40} \mathrm{Ca}$ ground state and for the levels of ${ }^{42} \mathrm{Sc}$ described below.

### 4.1. Wavefunctions

The wavefunction of the ${ }^{40} \mathrm{Ca}$ ground state contained ( $2 \mathrm{p}-2 \mathrm{~h}$ ) and ( $4 \mathrm{p}-4 \mathrm{~h}$ ) components in addition to the closed core ( $0 \mathrm{p}-0 \mathrm{~h}$ ) configuration. The full $170(2 \mathrm{p}-2 \mathrm{~h})$ states were considered that can be formed by raising two particles from the $d_{3 / 2}, s_{1 / 2}$ or $d_{5 / 2}$ singleparticle levels into the $\mathrm{f}_{7 / 2}, \mathrm{p}_{3 / 2}, \mathrm{p}_{1 / 2}$ or $\mathrm{f}_{5 / 2}$ states and coupling to $J=T=0$. The ( $4 \mathrm{p}-4 \mathrm{~h}$ ) component was projected from an intrinsic state formed by raising two protons and two neutrons from the highest Nilsson orbital in the sd shell into the lowest orbital in the fp shell. Minimizing the energy of this component with respect to the deformation parameter resulted in an oblate shape with the four particles occupying the $K=\frac{7}{2}-$ Nilsson orbital 10 and the four holes in the $K=\frac{1}{2}{ }^{+}$Nilsson orbital 11.

The shell-model hamiltonian diagonalized in this space gave the wavefunction for the $J^{\pi}=0^{+}$lowest energy states as

$$
a|0 \mathrm{p}-0 \mathrm{~h}\rangle+\sum_{i=1}^{170} b_{i}|2 \mathrm{p}-2 \mathrm{~h}\rangle+c|4 \mathrm{p}-4 \mathrm{~h}\rangle
$$

where the intensities of the components are $a^{2}=0.49, \Sigma_{i=1}^{170} b_{i}^{2}=0.51$ and $c^{2}=4 \times 10^{-5}$.
The ${ }^{42} \mathrm{Sc}$ states were composed of mixtures of $(2 \mathrm{p})$ and $(4 \mathrm{p}-2 \mathrm{~h})$ components. The neutron and proton of the ( 2 p ) configurations were allowed to occupy any singleparticle levels in the fp shell. For the ( $4 \mathrm{p}-2 \mathrm{~h}$ ) components, the two protons and two neutrons in the fp shell were placed in the $K=\frac{1}{2}^{-}$Nilsson orbital 14 and coupled to the two holes which were allowed to occupy any single-particle levels in the sd shell. The energy matrix for each angular momentum value was diagonalized for different sets of deformation parameters, those that minimized the energy of the lowest eigenfunction were used.

Versions 1 and 2 of this calculation were evaluated and in table 2 the predicted level energies, the intensities of ( 2 p ) and ( $4 \mathrm{p}-2 \mathrm{~h}$ ) parts of the wavefunctions and the intensity of the $\left(1 \mathrm{f}_{7 / 2}\right)^{2}$ component of the ( 2 p ) part are listed. Version 1 is identical with the ${ }^{42}$ Sc wavefunctions described by Flowers and Skouras (1969). A feature of this calculation was the low-lying levels of almost complete ( $4 \mathrm{p}-2 \mathrm{~h}$ ) nature that form a band based upon the $J^{\pi}=3^{+}$state at 1.17 MeV . Version 2 was prompted by the wish to associate this predicted $3^{+}$deformed state with a possible candidate, the $J^{n}=3^{(+)}$level found experimentally at 1.846 MeV . In Version 2 the $\mathrm{f}_{7 / 2}-\mathrm{d}_{3 / 2}$ splitting was increased from its previous value of 5.85 MeV to 6.15 MeV and the variational calculation repeated. A general upward shift of all $T=0$ levels was produced with the deformed $3^{+}$level occurring at 1.73 MeV .

Table 2. Component intensities of wavefunctions for the lowest $T=0$ states of ${ }^{42} \mathrm{Sc}$ predicted by versions 1 and 2 of the calculation

| State |  | Version 1 |  |  | Version 2 |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\left(\mathrm{f}_{7 / 2}\right)^{2}$ | Intensity (\%) |  | State |  |  | Intensity (\%) |  |
| Energy <br> (MeV) | $J^{\pi}$ |  | 2p | 2p-4h | Energy <br> ( MeV ) | $J^{\pi}$ | $\left(\mathrm{f}_{7 / 2}\right)^{\mathbf{2}}$ | 2 p | $4 \mathrm{p}-2 \mathrm{~h}$ |
| 0.34 | $7^{+}$ | 94.6 | 94.6 | 5.4 | 0.37 | $7{ }^{+}$ | 95.3 | 95.3 | 4.7 |
| 0.52 | $1{ }^{+}$ | 46.5 | 78.1 | 21.9 | 0.63 | $1^{+}$ | 50.9 | 85.9 | 14.1 |
| $1 \cdot 17$ | $3^{+}$ | 0.1 | 0.2 | 99.8 | 1.36 | $5^{+}$ | 57.8 | 94.0 | 6.0 |
| 1.32 | $5^{+}$ | 56.0 | 90.4 | 9.6 | 1.56 | $3^{+}$ | 49.9 | 85.0 | 15.0 |
| 1.51 | $3^{+}$ | 40.6 | 74.2 | 25.8 | 1.73 | $3^{+}$ | 1.8 | 3.0 | 97.0 |
| 1.64 | $1^{+}$ | 5.9 | 8.9 | 91.1 | 2.14 | $1^{+}$ | 3.5 | 4.9 | 95.1 |
| 1.74 | $4^{+}$ | 0.0 | 0.0 | 100 | 2.25 | $4^{+}$ | 0.0 | $0 \cdot 0$ | 100 |
| 2.13 | $3^{+}$ | 11.0 | 15.6 | 84.4 | 2.74 | $3^{+}$ | 4.4 | 5.1 | 94.9 |
| 2.47 | $5^{+}$ | 0.2 | 0.2 | 99.8 | 3.01 | $2^{+}$ | 2.2 | 3.6 | 96.4 |
| 2.54 | $1^{+}$ | 1.2 | 1.4 | 98.6 | 3.02 | $1^{+}$ | 0.8 | $1 \cdot 1$ | 98.9 |
| 2.54 | $2^{+}$ | 1.0 | 1.9 | 98.1 | 3.07 | $5^{+}$ | 0.2 | 0.2 | 99.8 |
| 2.99 | $6^{+}$ | 0.0 | 0.0 | 100 | 3.46 | $6^{+}$ | 0.0 | 0.0 | 100 |
| 3.09 | $2^{+}$ | 1.8 | 3.0 | 97.0 | 3.49 | $5^{+}$ | 37.8 | 95.6 | 4.4 |
| 3.18 | $5^{+}$ | 4.2 | 4.6 | 95.4 | 3.55 | $2^{+}$ | 7.8 | 12.2 | 87.8 |
| 3.20 | $3^{+}$ | 6.4 | 7.0 | 93.0 | 3.75 | $3^{+}$ | 21.6 | 42.8 | 57.2 |

### 4.2. DWBA analysis

The dWBa expression for the differential cross section to a ${ }^{42} \mathrm{Sc}$ level of spin $J$ can be written (Towner and Hardy 1969).
$\frac{\mathrm{d} \sigma}{\mathrm{d} \Omega} \alpha \frac{\mu_{\mathrm{x}} \mu_{\mathrm{d}}}{\left(2 \pi h^{2}\right)^{2}} \frac{k_{\mathrm{d}}}{k_{\alpha}}(2 J+1)$

$$
\times \sum_{\Lambda}^{\left[n_{1} l_{1} j_{1}\right]\left[n_{2} l_{2} j_{2}\right] L S J T}\left|b_{S T} \mathscr{S}^{1 / 2}\left(\left[n_{1} l_{1} j_{1}\right]\left[n_{2} l_{2} j_{2}\right] ; J T\right)\left(\begin{array}{ccc}
1 & 2 & L \\
\frac{1}{2} & \frac{1}{2} & S \\
j_{1} & j_{2} & J
\end{array}\right) B_{\Lambda}^{L}(\theta)\right|^{2}
$$

where $\mu_{\alpha}, \mu_{\mathrm{d}}$ and $k_{\alpha}, k_{\mathrm{d}}$ are the reduced masses and momenta of the $\alpha$ particle and deuteron respectively. The summation is over the quantum numbers of the transferred pair of particles and $b_{S T}=\delta_{S, 1} \delta_{T, 0}$. In the evaluation of $B_{\Lambda}^{L}(\theta)$ the form factor was calculated in the zero-range interaction approximation with the transferred particles bound in a Saxon-Woods well with spin-orbit coupling. The depth of the well was adjusted to make the nucleon binding energy equal to half the experimental separation energy of a neutron and proton from a ${ }^{40} \mathrm{Ca}$ core.

The optical potentials used for the $\alpha$ particles (W Makofske 1971 unpublished) and deuterons (Bassel et al 1964), used in the calculation of $B_{\Lambda}^{L}(\theta)$, were of the form

$$
-U(r)=\frac{V}{\left(1+\mathrm{e}^{x}\right)}+\frac{\mathrm{i} 4 W_{\mathrm{D}} \mathrm{e}^{y}}{\left(1+\mathrm{e}^{y}\right)^{2}}+U_{\text {Coulomb }}
$$

where $x=\left(r-r_{v} A^{1 / 3}\right) / a_{v}$ and $y=\left(r-r_{w} A^{1 / 3}\right) / a_{w}$. The values of the parameters used are listed in table 3.

Table 3. Optical model parameters used in the DWBA analysis

|  | $V$ <br> $(\mathrm{MeV})$ | $r_{v}$ <br> $(\mathrm{fm})$ | $a_{v}$ <br> $(\mathrm{fm})$ | $W_{\mathrm{D}}$ <br> $(\mathrm{MeV})$ | $r_{w}$ <br> $(\mathrm{fm})$ | $a_{w}$ <br> $(\mathrm{fm})$ | $r_{\text {Coul }}$ <br> $(\mathrm{fm})$ | $A$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\boldsymbol{\alpha}$ | 256 | 1.30 | 0.60 | 84.2 | 0.90 | 0.74 | 1.30 | 40 |
| d | 128 | 0.89 | 0.93 | 15.0 | 1.58 | 0.50 | 1.30 | 42 |

Spectroscopic amplitudes, $\mathscr{S}^{1 / 2}\left(\left[n_{1} l_{1} j_{1}\right]\left[n_{2} l_{2} j_{2}\right] ; J T\right)$ for the transferred particles $j_{1}, j_{2}$ were calculated from the wavefunctions described above by summing the amplitudes $M_{\mathrm{I}}, M_{\mathrm{II}}, M_{\mathrm{III}}$ and $M_{\mathrm{IV}}$ of the different transfer components illustrated in figure 2. Detailed expressions for these amplitudes are given below using a notation where $\left(j_{\alpha}, j_{\beta}\right) J_{1} T_{1}$ and $\left(j_{\gamma}^{-1}, j_{\delta}^{-1}\right) J_{1} T_{1}$ represent the two particles and two holes coupled to $J_{1} T_{1}$ in the ${ }^{40} \mathrm{Ca}(2 \mathrm{p}-2 \mathrm{~h})$ wavefunction; $\left(j_{\mu}, j_{v}\right) J T$ represent the two particles coupled to $J, T=0$ in the ${ }^{42} \mathrm{Sc}(2 \mathrm{p})$ wavefunction and ( 4 p ) $J_{2}$ and $\left(j_{k}^{-1} j_{\lambda}^{-1}\right) J_{3}$ represent the parts of the ${ }^{42} \mathrm{Sc}(4 \mathrm{p}-2 \mathrm{~h})$ wavefunction where the four particles are coupled to $J_{2}$ and the two


Figure 2. A schematic representation of the different contributions to the total two-particle transfer process.
holes to $J_{3}$. The expansion coefficients of the Nilsson particle $(k)$ and hole $\left(k^{\prime}\right)$ orbits into the spherical states $j_{x}$ and $j_{\beta}$ are given by $C_{j_{\alpha}}^{k}, C_{j_{\beta}}^{k^{\prime}}$ respectively. The expressions $M$ are then given by:

$$
\begin{aligned}
& M_{\mathrm{I}}= \delta_{j_{1} j_{\mu}} \delta_{j_{2} j_{v}} \\
& M_{\mathrm{H}}= \frac{-\delta_{J_{1} J} \delta_{T_{1} T} \delta_{j_{\alpha} j_{\mu}} \delta_{j_{\beta} j_{v}} \delta_{j_{1} j_{\nu}} \delta_{j_{2} j_{o}}}{\{(2 J+1)(2 T+1)\}^{1 / 2}} \\
& \begin{aligned}
M_{\mathrm{III}}= & (-1)^{J}+\tau \\
& \quad \delta_{J_{1} J_{3}} \delta_{T_{1} T} \delta_{j_{, j} j_{k}} \delta_{j_{j_{j} j_{\lambda}}} \frac{\left(2 J_{2}+1\right)^{1 / 2}}{\left\{(2 J+1)(2 T+1)\left(2 J_{3}+1\right)\right\}^{1 / 2}} \frac{N_{J_{2}} C_{j_{\alpha}}^{k} C_{j_{\beta}}^{k} C_{j_{1}}^{k} C_{j_{2}}^{k}}{\left(1+\delta_{j_{\alpha} j_{\beta}}\right)\left(1+\delta_{j_{1} j_{2}}\right)^{1 / 2}} \\
& \quad \times\left[\left\{1+(-1)^{T}\right\}^{2}\left(J 2 k J_{3}-2 k \mid J_{2} 0\right)\left(j_{1} k j_{2} k \mid J 2 k\right)\left(j_{x} k j_{\beta} k \mid J_{3} 2 k\right)\right. \\
& \quad+(-1)^{j_{2}+j_{\beta}\left\{1-(-1)^{J+T}\right\}\left\{1-(-1)^{J_{3}+T}\right\}\left(J 0 J_{3} 0 \mid J_{2} 0\right)\left(j_{1} k j_{2}-k \mid J 0\right)} \\
& \left.\quad \times\left(j_{\alpha} k j_{\beta}-k \mid J_{3} 0\right)\right]
\end{aligned}
\end{aligned}
$$

$$
\begin{aligned}
M_{\mathrm{IV}}=(-1)^{J+} & +\frac{N+1}{\left\{\left(2 J_{2}+1\right)\left(2 J_{3}+1\right)\right\}^{1 / 2}} \frac{N_{J_{2}}}{\left\{\left(1+\delta_{j_{k} j_{\lambda}}\right)\left(1+\delta_{j_{1} j_{2}}\right)\right\}^{1 / 2}} B_{J_{2}} C_{j_{k}}^{k^{\prime}} C_{j_{\lambda}}^{k^{\prime}} C_{j_{1}}^{k^{\prime}} C_{j_{2}}^{k^{\prime}} \\
& \times\left[\left\{1+(-1)^{T}\right\}^{2}\left(J 2 k^{\prime} J_{3}-2 k^{\prime} \mid J_{2} 0\right)\left(j_{1} k^{\prime} j_{2} k^{\prime} \mid J 2 k^{\prime}\right)\left(j_{k} k^{\prime} j_{\lambda} k^{\prime} \mid J_{3} 2 k^{\prime}\right)\right. \\
& +(-1)^{j_{2}+j_{\lambda}\left\{1-(-1)^{J+T}\right\}\left\{1-(-1)^{J_{3}+T}\right\}\left(J 0 J_{3} 0 \mid J_{2} 0\right)\left(j_{1} k^{\prime} j_{2}-k^{\prime} \mid J 0\right)} \\
& \left.\times\left(j_{k} k^{\prime} j_{\lambda}-k^{\prime} J_{3} 0\right)\right]
\end{aligned}
$$

where $N$ and $N_{J_{2}}$ are the normalizations of the ${ }^{40} \mathrm{Ca}(4 \mathrm{p}-4 \mathrm{~h})$ component and the fourparticle part of the ${ }^{42} \mathrm{Sc}(4 \mathrm{p}-2 \mathrm{~h})$ component respectively and $B_{J_{2}}$ is the overlap between the four-particle parts of the ${ }^{40} \mathrm{Ca}$ and ${ }^{42} \mathrm{Sc}$ wavefunctions.

## 5. Discussion

In table 4 a comparison is given between the measured strength of formation of ${ }^{42} \mathrm{Sc}$ levels and that predicted (i) by pure $\left(\mathrm{ff}_{7 / 2}\right)^{2}$ wavefunctions and (ii) by versions 1 and 2 of the calculations described above.

### 5.1. States of largely $\left(\mathrm{If}_{7 / 2}\right)^{2}$ structure

The relatively large differential cross sections measured for the $1^{+}+7^{+}$doublet at 0.61 MeV and the $3^{+}$and $5^{+}$levels at 1.491 MeV and 1.511 MeV confirm that these are the $T=0$ levels into which the $\left(1 \mathrm{f}_{7 / 2}\right)^{2}$ strength is mainly concentrated. Further evidence for the $\left(\mathrm{f}_{7 / 2}\right)^{2}$ configuration of the 1.491 MeV state is provided by the satisfactory explanation of the $B(E 2)$ value deduced from a recent lifetime determination for this level (Bertin et al 1972) by this structure.

In table 4 the predictions of each theoretical model were normalized on the $1^{+}+7^{+}$ doublet since the $7^{+}$level, which dominates the strength of the doublet, has almost pure $\left(1 \mathrm{f}_{7 / 2}\right)^{2}$ structure in each case. It is seen that for every calculation the relative strengths are qualitatively reproduced but a good quantitative representative is not provided by any version. While the experimental results show some collective enhancement for the $3^{+}$and $5^{+}$levels above that predicted by the $\left(1 \mathrm{f}_{7 / 2}\right)^{2}$ evaluation, versions 1 and 2 of the admixed calculation overestimate the effect.

### 5.2. States up to $2 \cdot 4 \mathrm{MeV}$ excitation

Experimental results on the spins and parities of states up to 2.4 MeV suggest the identification between experimental and predicted levels shown in table 4. The admixed calculations reproduce the large difference in strength between states of largely $\left(1 f_{7 / 2}\right)^{2}$ structure and other levels.

Detailed comparison of experimental and calculated strengths using this identification shows agreement to within a factor three for the experimental states at 1.846 MeV , $1.889 \mathrm{MeV}, 2.189 \mathrm{MeV}$ and 2.270 MeV . However the $1^{+}$level at 2.223 MeV is observed to be formed much more strongly than predicted. In the lifetime measurements of Roberson and Van Middelkoop (1971) it is seen that different gamma decay branches from this level give different lifetime values and the authors propose a second level within 5 keV of the known $1^{+}$state at this energy. An additional level which contributes quite strongly to the strength of the 2.223 MeV group in this experiment and has a principal gamma decay branch to the $1.587 \mathrm{MeV} 2^{+}$level (Roberson and Van Middelkoop 1971) can only be accounted for in the model considered by departing from the
Table 4. Compatison of experimettal and theoretical strengiths

identification of table 4 and assigning the $2^{+}$level at 2.54 MeV in version 1 to lie at $2.223 \pm 0.005 \mathrm{MeV}$. This new identification gives satisfactory predicted strengths of 6.5 and 9.7 for versions 1 and 2 of the calculation but to find a theoretical level to identify with the 2.270 MeV state it is necessary to go to the high energy of 3.09 MeV in version 1 or 3.55 MeV in version 2 .

The identification of the 1.846 MeV state with the head of a ( $4 \mathrm{p}-2 \mathrm{~h}$ ) deformed band is consistent with the comparison of strengths. However the $4^{+}, 5^{+}$and $6^{+}$members of the band are predicted to be so weakly excited ( $0.003,0.3$ and 0.002 au respectively) that they would fall below the threshold of detection of this experiment. It is clear that the comparatively strongly excited level at 2.391 MeV , tentatively assigned as $4^{+}$by Sherr et al, is not the $4^{+}$member of this band. Results on the 2.391 MeV level from gamma ray spectra taken in coincidence with protons from the ${ }^{40} \mathrm{Ca}\left({ }^{3} \mathrm{He}, \mathrm{p} \gamma\right)^{42} \mathrm{Sc}$ reaction ( $F$ M Nicholas 1969 unpublished) indicate that the main decay branch is to the $5^{+}$level at 1.511 MeV . Assuming the unlikelihood of octupole decay, this limits the spin value of the state to $3,4,5,6$ or 7 . In an attempt to identify this state with the predicted spectrum, the yields of all states with these spin values between 1.6 MeV and 3.2 MeV in version 1 were calculated assuming them to be the 2.391 MeV level. Only the $3^{+}$level predicted at 2.13 MeV provided sufficient strength, all others being too weak by factors of greater than 15 . However, if identification is made between the predicted $3^{+}$at 2.13 MeV and the observed 2.391 MeV state there is no candidate in the calculations for the experimental level at 2.189 MeV below 3.20 MeV in version 1 or 3.75 MeV in version 2 .

The difficulty in accounting for the states at 2.223 MeV and 2.391 MeV suggests that from an excitation of about 2.3 MeV in ${ }^{42} \mathrm{Sc}$ states of structure not accounted for by $2 p$ and $(4 p-2 h)$ admixtures begin to appear. These levels could be positive parity states with components of $(6 \mathrm{p}-4 \mathrm{~h}),(8 \mathrm{p}-6 \mathrm{~h})$ or $(4 \mathrm{p}(K=2)-2 \mathrm{~h})$ structure or the negative parity levels of ( $2 \mathrm{p}-1 \mathrm{~h}$ ) nature predicted by Dieperink and Brussaard (1968).

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